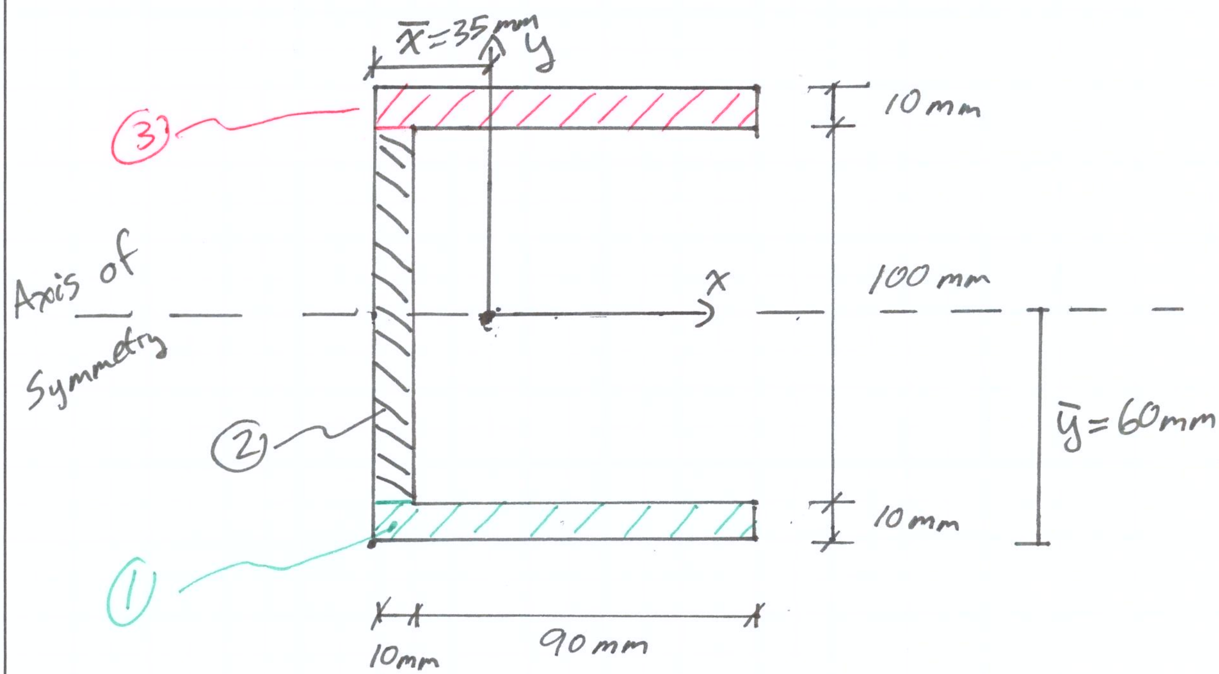
Find the centroid and I_x, I_y for the following shape.



- Idetify that this consists of 3 simple shape
- We know that the centroid lies on an axis of symmetry

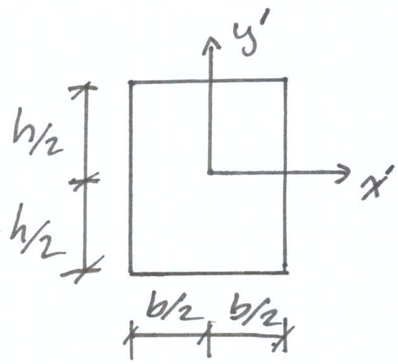
$$\bar{y} = \frac{1}{2}\text{ height} = \frac{(10+100+10)}{2} = \boxed{60\text{mm} = \bar{y}}$$

- We need the properties of each component to find $\bar{x}$, I_x, I_y

SHAPE	$A\ (mm^2)$	$\tilde{x}\ (mm)$ *	$\tilde{y}\ (mm)$ **
1	$100 \times 10 = 1000$	$100/2 = 50$	$-100/2 - 10/2 = -55$
2	$10 \times 100 = 1000$	$10/2 = 5$	0
3	1000	$\frac{100}{2} = 50$	$100/2 + 10/2 = 55$

* $\tilde{x}$ is relative to point $-(\bar{x})$ as defined in the figure. It is the extreme left edge of shape 2 at the intersection of the edge and the x centroidal axis

** $\tilde{y}$ is relative to the 'x' centroidal axis.



Location of centroid of a rectangle.

$$A_T = \Sigma A = 1000 + 1000 + 1000 = \boxed{A_T = 3000\,mm^2}$$

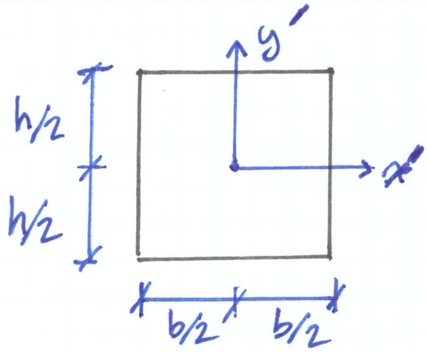
$$\bar{x} = \frac{\Sigma \tilde{x}_i A_i}{A_T} = \frac{50(1000) + 5(1000) + 50(1000)}{3000}$$

$$\boxed{\bar{x} = +35\ mm}$$

$$\bar{y} = \frac{\Sigma \tilde{y}_i A_i}{A_T} = \frac{-55(1000) + 0(1000) + 55(1000)}{3000} = \frac{0}{3000}$$

$$\boxed{\bar{y} = 0\ mm}$$

• We already knew this



$$\bar{I}_{x'} = \frac{1}{12} b h^3$$

$$\bar{I}_{y'} = \frac{1}{12} b^3 h$$

SHAPE	$\bar{I}_{x'}$ (mm^4)	$\bar{I}_{y'}$ (mm^4)	A (mm^2)	d_y (mm)	d_x (mm)	Ad_y^2 (mm^4)	Ad_x^2 (mm^4)
1	$\frac{1}{12}(100)(10)^3 = \frac{100\,000}{12}$	$\frac{1}{12}(100)^3(10) = \frac{10\,000\,000}{12}$	1000	-55	15	3,025,000	225,000
2	$\frac{1}{12}(10)(100)^3 = \frac{10,000,000}{12}$	$\frac{1}{12}(10)^3(100) = \frac{100\,000}{12}$	1000	0	-30	0	900,000
3	$\frac{1}{12}(100)(10)^3 = \frac{100\,000}{12}$	$\frac{1}{12}(100)^3(10) = \frac{10\,000\,000}{12}$	1000	55	15	3,025,000	225,000
Σ	850,000	1,675,000	3000			6,050,000	1,350,000

$$I_x = \Sigma\left(\bar{I}_{x'} + A d_y^2\right) = \Sigma \bar{I}_{x'} + \Sigma A d_y^2$$

$$I_y = \Sigma\left(\bar{I}_{y'} + A d_x^2\right) = \Sigma \bar{I}_{y'} + \Sigma A d_x^2$$

$$I_x = (850\,000) + (6,050,000) = 6,900,000 \text{ mm}^4$$

$$\boxed{I_x = 6.9 \times 10^6 \text{ mm}^4}$$

$$I_y = (1,675,000) + (1,350,000) = 3,025,000 \text{ mm}^4$$

$$\boxed{I_y = 3.025 \times 10^6 \text{ mm}^4}$$